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# Darrieus–Landau instability and Markstein numbers of premixed flames in a Hele-Shaw cell

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**Abstract**

Hydrodynamic flame instabilities are studied in a Hele-Shaw burner. By studying the development of perturbations, starting from a 2D Bunsen flame at the top of the burner, growth rates are measured for propane and methane–air mixtures, and compared to theoretical predictions. It is found that the dispersion relation in a Hele-Shaw cell has the same dependence with wavenumber $\sigma = k - k^2$ as the one predicted in tubes. Markstein numbers relative to fresh gases are obtained for propane and methane flames and compared to the literature.


*Keywords:* Premixed flames; Darrieus–Landau instability; Hele-Shaw burner; Markstein length

## 1. Introduction

Hydrodynamic flame instabilities can be important for turbulent burning, particularly at high pressure [1] . Even without turbulence, the propagation of a laminar flame in a tube can lead to a very complex non-linear evolution (these flames are often called self-turbulent) if the tube is large enough [2]. However it is difficult to study quantitatively the instability in three dimensions. It has therefore been suggested by Joulin and Sivashinsky in [3] to study quasi two dimensional flames in a Hele-Shaw cell. Ronney was the first to report [4] the experimental observation of flames in this configuration (see also the numerical simulation in [5]).

In the paper [6], qualitative results on self-turbulent flames in a Hele-Shaw burner were presented. We show in the present paper that these self-turbulent flames are caused by hydrodynamic instabilities which are very close to the ones observed in tubes.

The experiment, which will be described in more detail in [7], is summarized here: the Hele-Shaw burner consists of two glass plates 50 cm wide and 150 cm high, separated by a gap width of 4.7 mm. It is mounted vertically with the inlet gas mixture at the bottom, open at the upper end. The equivalence ratio $\phi$ of the premixed gas is regulated by mass-flow controllers Bronkhorst EL-Flow. A 2D

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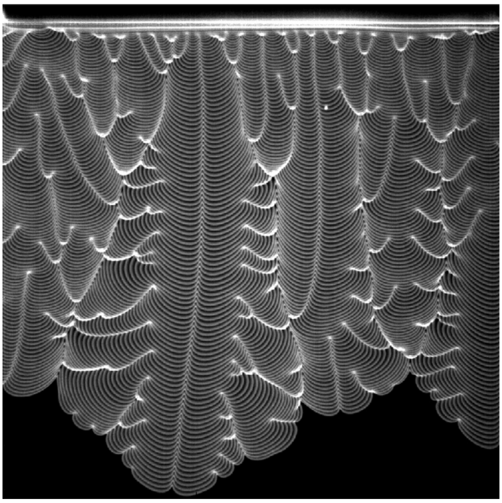

Fig. 1. Spatio temporal evolution ($50 \times 50$ cm) of a propane–air flame with equivalence ratio $\phi = 0.81$.

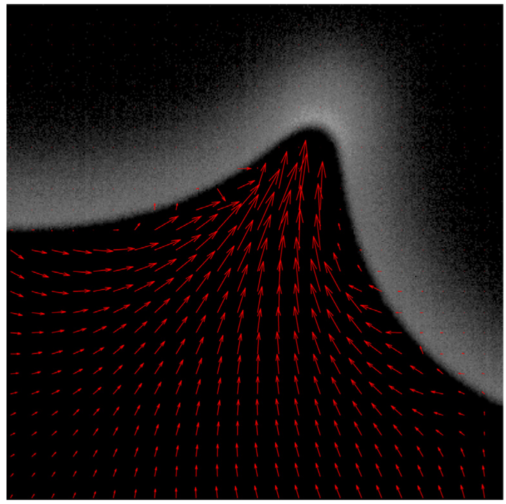

Fig. 2. Velocity field for a propane–air flame with equivalence ratio $\phi = 0.81$.

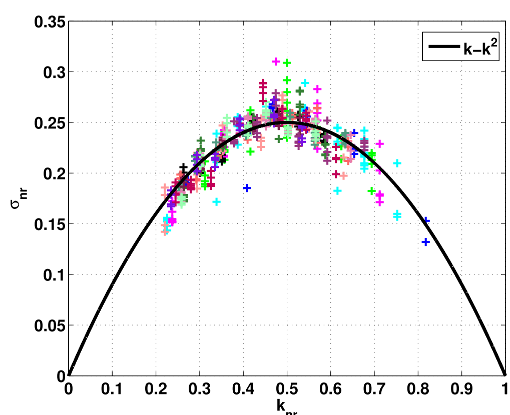


Fig. 3. Normalized growth rate $\sigma_{nr} = \sigma/4\sigma_{max}$ versus normalized wavenumber $k_{nr} = k/2k_{max}$ for different mixtures (corresponding to different colors) of methane and propane. The black curve is the function $k - k^2$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

Bunsen flame is ignited at the top of the burner where it remains anchored thanks to a flow rate in excess. Closing the valve at the bottom of the burner stops the flow and the downwards flame propagation is recorded using a high-speed video camera. The initial Bunsen flame is forced at a certain wavelength by using a plate with periodic indentation at the top of the burner. A typical situation, including the linear and non linear stage of the evolution is shown in Fig. 1. As seen in this figure, this very large burner contains a lot of unstable modes, a situation typical of applications using premixed flames at high pressure. In the rest of the paper, we will show that by looking at the linear stage of this figure close to the top of the burner, growth rates can be measured, for propane and methane–air flames, leading to a quantitative analysis of the instability, which is usually difficult to obtain in three dimensions. However, although the quasi two dimensional character of the flame in the Hele-Shaw burner leads to a much easier image analysis, there is the obvious drawback that the flame is affected by heat and viscous losses. We provide in Fig. 2 a qualitative illustration that the flame in the burner is essentially subjected to the usual Darrieus–Landau instability. In this figure, the velocity field obtained by PIV measurements close to one of the cusps seen in Fig. 1 is shown. This velocity field is typical of a Darrieus–Landau instability seen from a reference frame where the flame is advancing (this figure can be compared to the streamlines obtained by numerical simulation in [1] for instance) The purpose of the present paper is to present the quantitative results we get in this burner for the growth rates of the hydrodynamic instability. We will even use these results in order to get indirect measurements of the Markstein number in the Hele-Shaw cell.

Before discussing the different curves of propane and methane flames let us first try to collapse our growth rates measurements by using as reference values the maximum growth rate $\sigma_{max}$ and the wavenumber corresponding to this maximum growth rate $k_{max}$. In 1982 Frankel and Sivashinsky [8], Pelcé and Clavin [9] and Matalon and Matkowsky [10], obtained a dispersion relation, valid for small wavenumbers, for constant $\rho D$ ($\rho$ is the density, $D$ a diffusion coefficient) which was found to be of the form $\sigma \propto k - k^2$. We show in Fig. 3 that all our measurements collapse on

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this type of curve, leading to a more quantitative confirmation that the instability in the Hele-Shaw burner is still a Darrieus-Landau instability. The values of the parameters of Fig. 3 are given in the next sections. In the propane–air case, the equivalence ratio where we were able to measure growth rates is limited to a range $\phi = 0.81$ to $\phi = 1$, so it cannot be excluded that a thermodiffusive instability could be present for rich propane flames. It must be noted that in the hydrogen–air case, it has been shown only recently by a direct numerical simulation with complex chemistry, that a thermodiffusive instability ($\sigma \propto k + k^2 - k^4$, the pure thermodiffusive instability without Darrieus–Landau terms being $\sigma \propto k^2 - k^4$) is present for very lean flames [11].

## 2. Modified Clavin–Garcia dispersion relation

We have found in the previous section that the dispersion relation for various mixtures of methane and propane with air is of the form $\sigma \propto k - k^2$. We would like however to study slow flames, thus taking into account the Froude number. A dispersion relation valid for temperature-dependent diffusivities (as in most indirect measurements of Markstein lengths [12,13], we limit ourselves to the following temperature dependence: $\rho D \propto \sqrt{T}$, $T$ being the temperature) has been proposed by Clavin and Garcia [14] (see also [13]) in the form of a quadratic equation:

$$A(k)\sigma^2 + B(k)\sigma + C(k) = 0 \qquad (1)$$

With $A(k)$, $B(k)$ $C(k)$ coefficients depending on gas expansion $E = \rho_u/\rho_b$, Markstein number $Ma$, Froude number $Fr = u_l^2/g\delta_l$ (where $u_l$ is the laminar flame speed and $\delta_l$ the flame thickness, $g$ is the acceleration of gravity) and Prandtl number $Pr$:

$$\begin{aligned} A(k) &= \frac{E+1}{E} + \frac{E-1}{E}k\left(Ma - J\frac{E}{E-1}\right) \\ B(k) &= 2k + 2Ek^2(Ma - J) \\ C(k) &= \frac{E-1}{E}\frac{k}{Fr} + (E-1)\frac{k^3}{k_m} \\ &- (E-1)k^2\left[1 + \frac{1}{EFr}\left(Ma - J\frac{E}{E-1}\right)\right], \end{aligned} \qquad (2)$$

where $k_m^{-1} = \sqrt{E} + \frac{3E-1}{E-1}Ma - \frac{2E}{E-1}J + (2Pr - 1)H$. The integrals $J$ and $H$ can be found in [13,14].

This dispersion relation has the advantage of including explicitly the Froude number, but for very unstable flames, it leads for large values of $k$ to very asymmetrical dispersion relations (see [13] for instance). We have seen in Fig. 3 that on the contrary, the experimental dispersion relation is very symmetrical.

Anyway the theoretical dispersion relations are only valid for low values of $k$, so we search for a

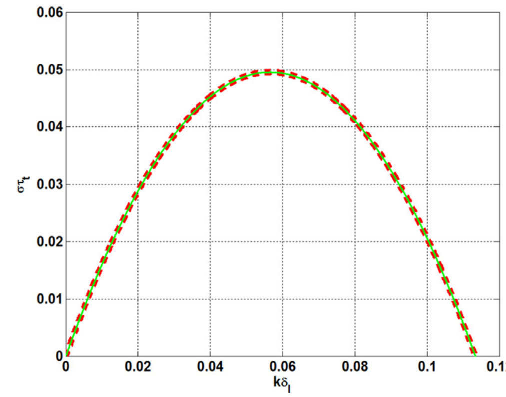


Fig. 4. Comparison between the modified Clavin–Garcia model without gravity (green continuous curve) with the work of Altantzis et al. (red dashed curve), $E = 7.9$, $U_l = 39.5$ cm/s and $Ma = 3.69$. The curves are superimposed. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

development of the Clavin–Garcia dispersion relation, valid for small $k$ and $1/Fr = O(k)$

$$\sigma = a_g k + b_g k^2 + O(k^3) \qquad (3)$$

where coefficients $a_g$ et $b_g$ are functions of $k$ including Froude number effects:

A straightforward development leads to

$$a_g = \frac{E}{E+1}\Big[S - 1\Big] \qquad (4)$$

where $S = \sqrt{1 + E - \frac{1}{E} - \frac{E^2-1}{E^2 Frk}}$, the relation $\sigma = a_g k$ being the known [15] Darrieus–Landau result with gravity, but without Markstein number effect, and

$$\begin{aligned} b_g = \Bigg[&\left(-a_g^2\frac{E-1}{E} - 2a_gE + \frac{E-1}{EkFr}\right)Ma \\ &+ J\left(a_g^2 - \frac{1}{kFr} + 2a_gE\right) - \frac{E-1}{k_m}\Bigg]/2S \end{aligned} \qquad (5)$$

The same type of dispersion relation has been obtained by Matalon and coworkers for temperature dependent diffusivities, in the case without gravity (see [16] for the derivation, the correct results are to be found in Altantzis et al. [17]). Figure 4 shows that the modified Clavin–Garcia relation agrees with Ref. [17] for the case without gravity.

Furthermore, it should be noted that in the region of vanishingly small wavenumbers, the gravity effects lead to complex values of the growth rate whose small $k$-expansion has to be sought now in powers of $k^{1/2}$. A straightforward calculation shows that the real part is still linear in $k$ but with a negative slope whereas the imaginary part is proportional to $k^{1/2}$. Anyway, in the present experimental

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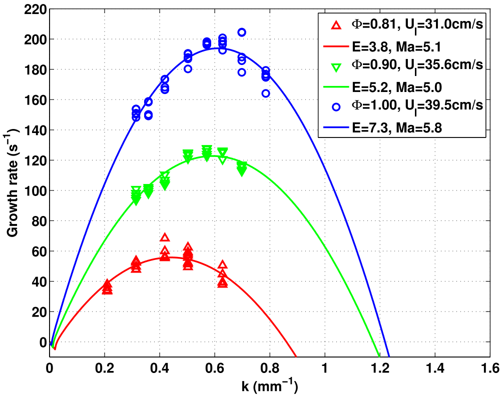


Fig. 5. Growth rates of propane-air mixtures at different equivalence ratios compared to the modified Clavin–Garcia relation.

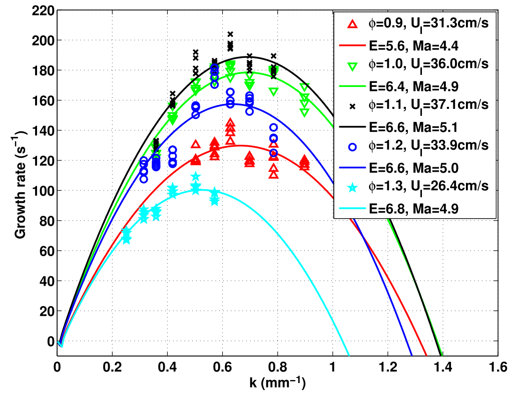


Fig. 6. Growth rates of methane–air mixtures at different equivalence ratios compared to the modified Clavin–Garcia relation.

conditions, this region appears to be of negligible extent.

We will use the modified Clavin Garcia dispersion relation in the rest of the paper, taking as units the thermal flame thickness $\delta_l = D_{th}/u_l$ and the transit time $\tau_t = \delta_l/u_l$. The laminar flame speeds $u_l$ to be used will be the values measured by Bosschaart and de Goey by the heat flux method [18] for propane and methane air flames. Note that we have not taken into account that the flame is curved in the thickness, which could lead to a larger effective propagation velocity. Unfortunately Joulin and Sivashinsky [3], the available theory in a Hele-Shaw cell neglects Markstein lengths effects and supposes that the flame is flat in the thickness, which is not the case in the experiment, where the flame adopts a convex shape as a result of Darrieus–Landau instability and the lateral quenching on the cell plates. This is partially counterbalanced by stretch and by the back slope of temperature due to the burnt gas cooling. Our experiments show a net increase of the flame propagation velocity, except for lean flames, and the effects on the instability appears to be reduced just to quantitative modifications (ca. 30% reduction on the growth rates, see [7]). This could show that the transverse directions are independent.

## 3. Experimental results for propane and methane–air mixtures

In this section, we will present our measures of growth rates of propane and methane–air flames compared to a fit using the modified Clavin–Garcia formula of the previous section. Errors caused by the transient observed after closing the valve are discussed in the supplemental material. Figure 5 shows the case of propane–air flames for the three equivalence ratios $\phi = 0.81$, $\phi = 0.9$ and $\phi = 1.0$. As in previous experiments with propane [13,19] the range of equivalence ratios where it was possible to measure growth rates is limited. For low equivalence ratios, we are close to the extinction limit for the Hele-Shaw cell with a gap width of 4.7 mm, which probably explains the low growth rates observed for $\phi = 0.81$. The maximum growth rate for $\phi = 1.0$ is approximately more than three times larger than for $\phi = 0.81$, and more than 50% larger than the $\phi = 0.9$ case. This rapid variation can be seen in the effective expansion ratios reported on the figure, which also shows the Markstein length given by a two-parameter ($E$ and $Ma$) fit with Eq. (3), as well as the laminar flame speeds used in the fit. The effective experimental expansion ratio $E = 7.3$ for $\phi = 1.0$ can be compared to the value obtained by Cantera with a San Diego mechanism $E = 7.95$. Unfortunately we were not able to measure growth rates for rich propane flames, as explained in [7] (because of difficulties to anchor the flame for slightly rich flames, or Bunsen flame already cellular for very rich flames).

In the methane case (Fig. 6), the accessible range of equivalence ratio was larger, from $\phi = 0.9$ to $\phi = 1.3$. Starting from lean flames, the maximum growth rate increases up to $\phi = 1.1$. the value of expansion ratio and Markstein lengths are also given, for the stoichiometric case, we obtain $E = 6.4$ for $\phi = 1.0$, with a difference larger than in the propane case compared to the value obtained by Cantera with a Grimech 3.0 mechanism $E = 7.47$.

## 4. Indirect measurement of Markstein numbers

We compare in this section the Markstein lengths measured for propane and methane air mixtures, with the existing values in the literature. Let us recall that the Markstein lengths we discuss are Markstein lengths relative to fresh gases,

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with less measurements than the burnt gases Markstein length often obtained in the spherical flame configuration. Our measurements are indirect measurements, the Markstein length is determined here by a comparison with a theoretical formula. In our case we recall that the theory predicts a growth rate of the form $\sigma \propto k - k^2$, a two-parameter fit of the experimental dispersion relation allows the determination of the coefficient of the $k$ term which depends on gas expansion, and of the coefficient of the $k^2$ term which depends on the Markstein number. Other indirect measurements exist. Searby and Quinard [12] reported Markstein lengths at the stability threshold for very slow flames (methane and propane) highly diluted with nitrogen, Clanet and Searby [19] used a parametric restabilization of the plane front by an acoustic field to measure growth rates at one wavelength for very lean propane flames, Truffaut and Searby [13] studied the development of perturbations on oblique flames for rich propane–air mixtures.

Our values will also be compared to direct measurements of Markstein lengths. These direct measurements are often performed in conditions where one component of the total stretch, strain or curvature, dominates. Durox et al. [20], have obtained values dominated by curvature by studying imploding cylindrical flames in the propane and methane case. Another experiment dominated by curvature is Garcia Soriano et al. [21] who studied rich Bunsen burner methane flames.

Most other direct measurements are dominated by strain. Clavin and Joulin in 1989 [22] (see also Clavin and Graña-Otero [23]) showed that in a strain dominated case, there is an important difficulty. If the velocity used in the measurement corresponds to a low temperature, the Markstein length obtained has a tendency to be negative [24–26], contrary to most indirect or curvature dominated values. In [22,23] the following procedure was suggested, first extrapolate the velocity field toward the reaction zone (the maximum of heat release for instance) to determine the Markstein length. Here we only present strain dominated values using extrapolation , the experiment of Deshaies and Cambray [27] and the numerical simulation with complex chemistry of Davis et al. [28] for propane and methane. Let us note however that the following correction has been proposed (see [23]) for non extrapolated strain Markstein numbers: if the Markstein number is measured at a distance $\delta x_f$ from the flame in the fresh gases, simply add $\delta x_f / \delta_l$ to the reported value. As seen in ([25] Fig. 12), where a typical $\delta x_f$ is shown, this would generally lead to positive Markstein numbers.

Figure 7 shows the Markstein numbers in the propane case. The present measurements correspond to the black curve for the three equivalence ratios shown in Fig. 5. The last point showing

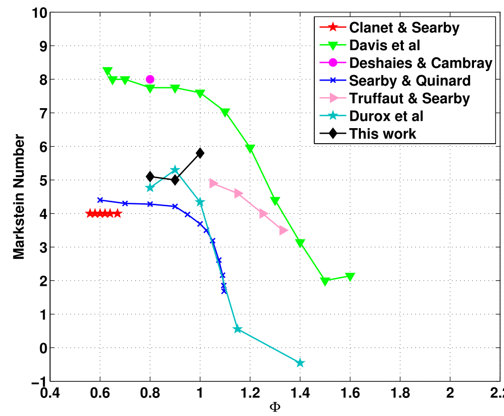


Fig. 7. Markstein numbers for propane flames versus equivalence ratio.

an increase of the Markstein number is possibly caused by experimental errors, a discussion of the interpolation error in the Markstein number measurement can be found in the supplemental material. Other indirect measurements are Truffaut and Searby (pink curve), Clanet and Searby (red) and Searby and Quinard (blue). Durox et al. (light blue) shows direct measurements of Markstein length in a case dominated by curvature. Strain dominated values are Deshaies and Cambray (magenta, only one point) and Davis et al. (green curve).

Let us first discuss the indirect measurements. The present values are in reasonable agreement with those of Truffaut and Searby for rich flames, but are above the Markstein numbers of Searby and Quinard and of Clanet and Searby. However the Searby and Quinard results concern flames diluted with a lot of nitrogen. As the gas expansion increases the Markstein length, it is perfectly normal that their Markstein numbers are lower than our values or Truffaut Searby values. The low values of Clanet and Searby without dilution are more surprising, but should probably be taken with care, their method of using an acoustic field to stabilize the plane flame leads to an important initial acceleration which could cause problems.

Let us turn to the direct measurements. If Deshaies and Cambray and Davis et al. seem to agree, the curvature values of Durox et al. are typically much lower than the values dominated by strain (the indirect measurements being somewhere in between). Different explanations of this discrepancy are possible, we do not have many experiments, some experiments could have a huge error bar, for instance the extrapolation in the strain rate dominated values could lead to Markstein numbers too high. Another possible explanation has been proposed by Clavin and Graña-Otero [23], the

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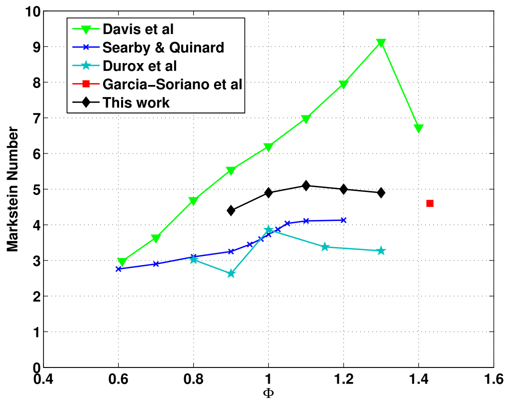


Fig. 8. Markstein numbers for methane flames versus equivalence ratio.

values of Markstein lengths relative to strain and curvature could simply be different, a possibility also suggested in a recent paper by Thiesset et al. [29] (see also the model with volumetric heat losses of Nicoli and Clavin [30]).

In Fig. 8 is presented the methane case. As before, our Markstein numbers correspond to the black curve, Searby and Quinard to the blue curve, Durox et al. to the light blue curve, Davis et al. to the green curve, Garcia-Soriano et al. [21] to the red point for a very rich methane flame. As expected because of the different dilutions our Markstein numbers are above those measured by Searby and Quinard. As in the propane case, the curvature dominated Markstein numbers are much lower than the strain dominated Davis et al. values, our indirect values are still in between. There is of course the difference with the propane case that for the methane-air mixtures, the Markstein numbers are lower for lean flames, the exact opposite of the situation for propane.

## 5. Conclusion

In this paper, we have reported values of growth rates of the Darrieus–Landau instability in a Hele-Shaw burner. Although heat losses lead to lower growth rates than in a tube, we were surprised to see that the Darrieus–Landau instability in the cell is essentially of the same type as in a tube. Using a fit with a theoretical dispersion relation (a modified Clavin–Garcia dispersion relation, introduced here) we were able to obtain indirect measurements of Markstein numbers relative to fresh gases, which compare reasonably well to the indirect measurements of Markstein numbers in the literature. As recalled here, the discrepancy with direct measurements is much higher.

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## Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.proci.2018.05.030.

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